ROUND I: Combinatorics

ALL ANSWERS MUST BE IN SIMPLEST EXACT FORM AS POSITIVE INTEGERS

1. How many 2-digit even whole numbers have an odd number as the sum of their digits?

2. Ten people are running for chair and vice-chair of a club. After they are selected three directors will be elected from the remaining candidates. In how many different ways can these offices be filled?

3. A vending machine dispenses products that cost 50 cents each. It will accept only nickels, dimes and quarters and requires exact change. How many different combinations of coins must the machine be able to accept as payment for one item?
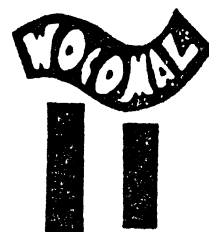
ANSWERS
(1 pt) 1._____

(2 pts) 2._____

(3 pts) 3._____

Algonquin, Shepherd Hill, West Boylston

ROUND II: Algebra I - open

ALL ANSWERS MUST BE IN SIMPLEST EXACT FORM

1. Write $\dfrac{2^{40}}{4^{20}}$ in simplest form.

2. Solve $3^{n+2} = 3^n + 1944$ for n

3. The cost to a concessionaire at a ball game is the sum of a fixed part and a part that varies directly as the number of people in attendance. If 4000 people attend a game, his cost will be $1300. If 2800 people attend, his cost will be $970. Find his cost if 1000 people attend.

ANSWERS
(1 pt) 1._____

(2 pts) 2._____

(3 pts) 3.__$_____

Hudson, South, Tantasqua

ROUND III: Logarithms, exponents, and radicals

ALL ANSWERS MUST BE IN SIMPLEST EXACT FORM

Solve each equation for x

1. $9 + 2\sqrt{4+3x} = 29$

2. $\left(\dfrac{1}{3}\right)^{-1} \cdot 9^{x-1} = \left(\dfrac{1}{9}\right)^{2x-1}$

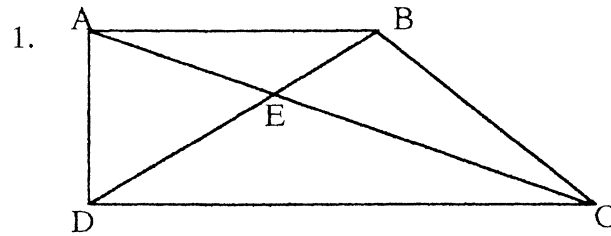3. $(\log_a x)^2 = \log_a(a^2 x)$

ANSWERS
(1 pt) 1._____

(2 pts) 2._____

(3 pts) 3._____

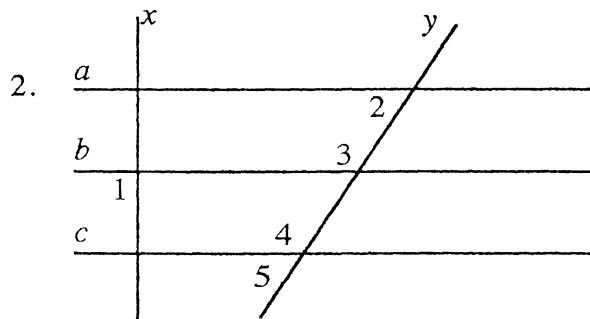Bromfield, Hudson, Shepherd Hill, Shrewsbury, Worcester Academy

ROUND IV: Parallel Lines and Polygons

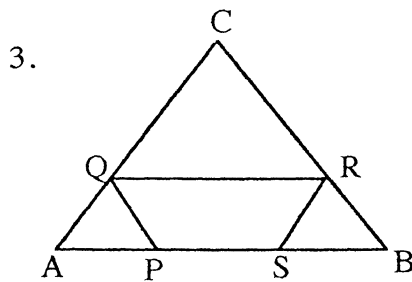ALL ANSWERS MUST BE IN SIMPLEST EXACT FORM

1.

Given: $\overline{AB} \parallel \overline{CD}$, $\overline{AD} \perp \overline{DC}$,

   $m\angle BAC = 32°$, and $m\angle BDC = 27°$

Find: $m\angle DEC$

2.

Given: $a \parallel b \parallel c$ and $x \perp a$

Find: $3m\angle 1 + 2m\angle 2 - 4m\angle 3$

   $- m\angle 5 + 5m\angle 4$

3.

Given: Equilateral Triangle ABC , AC = 20,

   $\overline{PQ} \parallel \overline{BC}$, $\overline{QR} \parallel \overline{AB}$, $\overline{RS} \parallel \overline{CA}$, and

   $PQ + QR + RS = 28$

Find: PS

ANSWERS

(1 pt)   1._____

(2 pts) 2._____

(3 pts) 3._____

Auburn, Holy Name, Shrewsbury

ROUND V: Analytical Geometry of straight lines and conic sections

ALL ANSWERS MUST BE IN SIMPLEST EXACT FORM

1. Find the least distance between the graphs of $x^2+y^2 = 1$ and $(x + 5)^2 + (y + 12)^2 = 1$.

2. How many distinct points are common to the graphs of $x^2 + 4y^2 = 1$ and
    $$4\left(x - \frac{1}{2}\right)^2 + y^2 = 1 \quad ?$$

3. The line $3x - 4y = 6$ intersects the graph of $|x| + |y| = 9$ in two points. Find the sum of the x-coordinates of these two points, Give an integer, mixed number, or fraction answer, not a decimal.

ANSWERS
(1 pt) 1._____

(2 pts) 2._____

(3 pts) 3._____

Bromfield, St. John's, South, Tahanto

TEAM ROUND: Topics of Previous rounds and open

ALL ANSWERS MUST BE IN SIMPLEST EXACT FORM AND ON THE
    SEPARATE TEAM ANSWER SHEET   2 points each

1.   A positive integer is placed at each vertex of an octagon which has one vertex already
     labelled 1. Consecutive vertices must be labelled with numbers that differ by 1. In how many
     different ways may the other seven vertices be labelled?

2.   Specify all values for m that make the value of $\dfrac{2}{4-m}$ greater than 1000.

3.   Solve for x : $\log_5(5\log_5(\log_5 x^{-5})) = 1$

4.   An equilateral triangle and a regular hexagon have equal perimeters. If the area of the triangle
     is 2, find the area of the hexagon.

5.   Three vertices of a parallelogram are (1,1), (3,5), and (-1,4). Find all possible ordered pairs
     that could be coordinates of the fourth vertex.

6.
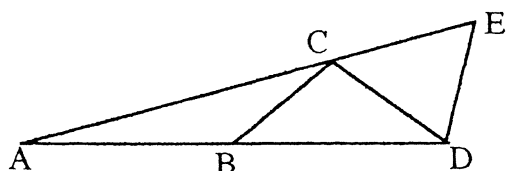
If AB=BC=CD=DE and m$\angle$ADE = 140°, find
m$\angle$A. (Diagram not to scale, but there are two sets
of collinear points.)

7.   Two candles are 36cm tall. After being lit, one candle lasts 9 hours and the other lasts 12
     hours. If both candles are lit at the same time, in how many hours will one candle be twice as
     tall as the other?

8.   Solve $(x^2 - 6x + 4)^2 = 16$

9.   If Peter's Pizza uses 12 ounces of pizza dough for a 14-inch pizza, how many ounces of
     dough will it need for an 18-inch pizza? Round your answer to the nearest integer. A 14-inch
     pizza has a diameter of 14 inches. The pizza crust thickness is the same in both.

Auburn, Bartlett, Burncoat, Hudson, Mass. Academy, South, Westborough